UNIT-4

STATISTICAL INFERENCE

TESTS OF HYPOTHESIS

Statistical Inference: **Introduction, Hypothesis and Hypothesis testing, Directions and errors in hypothesis testing, parametric Vs. Non-parametric tests, Hypothesis testing of Population Parameters with Large Samples (Z-test), with small samples (t-test), Hypothesis testing based on F-distribution (F-test), Chi-Square test.**

Course Outcome: Analyse the sample data and interpret the results through hypothetical testing methods for valid conclusions.

Chapter Outlines:

- Null and Alternate Hypothesis, Type-I and Type-II errors
- Test the significance difference between sample means and hypothetical means using Z-test
- Test the significance difference between sample proportions and hypothetical proportions using Z-test
- Tests of Hypothesis of means based on t-test
- Test of Hypothesis of Variances based on F-test
- χ^2-test for Goodness of fit and Independence of attributes.

TESTS OF HYPOTHESIS

LARGE SAMPLE TESTS ($n \geq 30$)

1) Z-test for single Mean
2) Z-test for difference of Means
3) Z-test for single Proportion
4) Z-test for difference of Proportions

SMALL SAMPLE TESTS ($n<30$)

1) t-test for single Mean
2) t-test for difference of Means
3) Paired t-test
4) F-test for Variances
5) χ^2 - test for Goodness of fit
6) χ^2 - test for Independence of Attributes

UNIT-4

INTRODUCTION

Hypothesis testing is a statistical method that involves testing an assumption about a population parameter. It's a powerful tool that can be used in management to evaluate the plausibility of a hypothesis.

HYPOTHESIS :

A statistical hypothesis is a statement or assertion about a population on the basis of information available from a sample.

NULL HYPOTHESIS :

Any statistical hypothesis which is defined as statement of no difference, is called Null Hypothesis, which is usually denoted by H_0 .

For example H_0 : $\mu = \mu_0$.

ALTERNATIVE HYPOTHESIS :

Any hypothesis or statement which is against or complementary to the null hypothesis is called an alternative hypothesis, usually denoted by H_1.

For example (i) $H_1 : \mu \neq \mu_0$ (Two tailed test)

(ii) $H_1 : \mu > \mu_0$ (Right tailed test)

(ii) $H_1 : \mu < \mu_0$ (left tailed test)

(ii) and (iii) statements are also known as one tailed tests.

CRITICAL REGION :

A region (corresponding to a statistic t) in the sample space S which amounts to rejection of H_0 is termed as critical region or region of rejection.

ONE TAILED AND TWO TAILED TESTS :

In any test the critical region is represented by a portion of the area under the probability curve of the sampling distribution of the test statistic.

A test of any statistical hypothesis where the alternative hypothesis is one tailed (right tailed or left tailed) is called a *one tailed test*.

(i) $H_1 : \mu > \mu_0$ (Right tailed test) or (ii) $H_1 : \mu < \mu_0$ (left tailed test)

A test of statistical hypothesis where the alternative hypothesis is two tailed is called a *two tailed test.* $H_1 : \mu \neq \mu_0$ (Two tailed test)

TYPE-I AND TYPE-II ERRORS :

The decision to accept or reject the null hypothesis H_0 is made on the basis of the information supplied by the observed sample values. The four possible situations that arise in any test procedure are

(i) **Rejecting a lot (H_0), when it is good**.

(ii) Accepting a lot (H_0), when it is good.

(iii) Rejecting a lot (H_0), when it is not good.

(iv) **Accepting a lot (H_0), when it is not good.**

The error of **Rejecting a lot (H_0) when it is good is** called Type-I error.

The error of **Accepting a lot (H_0) when it is not good** is called Type-II error.

Level of significance (α):

The probability of type-I error is known as the level of significance of the test. It is also called the size of the critical region.

α= p (Type-I error)

=p(Rejecting a lot (H_0), when it is good)

POWER OF THE TEST (1-β):

The probability of type-II error is denoted by β , i.e., β=p(type-II error).

Then 1-β is defined as the probability of **Accepting H_0 when it is good** is called the power of the test.

PARAMETRIC VS NON-PARAMETRIC TESTS

What is a Parametric Test?

In Statistics, the generalizations for creating records about the mean of the original population is given by the parametric test. This test is also a kind of hypothesis test. A t-test is performed, and this depends on the t-test of students, which is regularly used in this value. This is known as a parametric test.

The t-measurement test hangs on the underlying statement that there is the ordinary distribution of a variable. Here, the value of mean is known, or it is assumed or taken to be known. The population variance is determined in order to find the sample from the population. The population is estimated with the help of an interval scale and the variables of concern are hypothesized.

What is a Non-Parametric Test?

There is no requirement for any distribution of the population in the non-parametric test. Also, the non-parametric test is a type hypothesis test that is not dependent on any underlying hypothesis. In the non-parametric test, the test depends on the value of the median. This method of testing is also known as distribution-free testing. Test values are found based on the ordinal or the nominal level. The parametric test is usually performed when the independent variables are non-metric. This is known as a non-parametric test.

Differences Between Parametric Test and Non-Parametric Test

Properties	**Parametric Test**	**Non-Parametric Test**
Assumptions	Yes, assumptions are made	No, assumptions are not made
Value for central tendency	The mean value is the central tendency	The median value is the central tendency
Correlation	Pearson Correlation	Spearman Correlation
Probabilistic Distribution	Normal probabilistic distribution	Arbitrary probabilistic distribution
Population Knowledge	Population knowledge is required	Population knowledge is not required
Used for	Used for finding interval data	Used for finding nominal data
Application	Applicable to variables	Applicable to variables and attributes
Examples	T-test, z-test	Mann-Whitney, Kruskal-Wallis

PARAMETRIC TESTS

PROCEDURE FOR TESTING OF HYPOTHESIS

From the problem context, identify the parameter of interest and then follow the steps listed below

Step1: State the Null Hypothesis H_0

Step2: Specify an appropriate Alternate Hypothesis H_1

Step3: Choose the Level of Significance α

Step4: Calculate the appropriate test statistic value

Step5: Compare the Calculated test statistic value with Critical region value at α level

Step6: Conclusion:- Reject H_0 when the calculated test statistic value is greater than the critical region value otherwise we fail to Reject H_0

CRITICAL VALUES OF Z

	Level of significance		
	1 %	**5 %**	**10 %**
Two - tailed	**2.58**	**1.96**	**1.645**
One - tailed	**2.33**	**1.645**	**1.28**

LARGE SAMPLE TESTS ($n \geq 30$)

1. Z - test for SINGLE MEAN

TEST OF SIGNIFICANCE FOR SINGLE MEAN

Let us consider $x_1, x_2, x_3, \ldots x_n$ is a random sample of size n taken from a normal population with mean μ and variance σ^2 and let $\bar{x}$ is the mean of this sample also follows a normal distribution with mean μ and variance $\frac{\sigma^2}{n}$ then to test the significance difference between the sample mean $\bar{x}$ and a specified population mean μ_0 (or to test whether the sample has been drawn from a normal population of mean μ_0)

That is for testing $H_0 : \mu = \mu_0$ (Null Hypothesis)

then the test statistics is given by $$Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$ (under H_0)

Conclusion: If the calculated value of $|Z|$ is greater than the table value of Z at the level of significance α then we reject H_0 otherwise we accept H_0.

Example-1: An ambulance service claims that it takes on the average 8·9 minutes to reach its destination in emergency calls. To check on this claim, the agency which licenses ambulance services has them timed, on 50 emergency calls, getting a mean of 9·3 minutes with a standard deviation of 1·6 minutes. What can they conclude at the level of significance 0·01?

Solution: Given that

$specified\ (True)\ Mean\ \mu = 8.9,\quad standard\ \ deviation\ \ \sigma = 1.6$

$$sample\,size\ \ n = 50,\ \ sample\,mean\ \bar{x} = 9.3$$

we have to test

$Null\ Hypothesis \qquad H_0 : \mu = 8.9$

$Alternate\ \ Hypothesis \quad H_1 : \mu \neq 8.9\ (Two-tailed\ \ test)$

$Level\ \ of\ \ significance \quad \alpha = 0.01$

$$Test\ \ statistic\ Z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{9.3-8.9}{1.6/\sqrt{50}} = \frac{0.4}{0.2263} = 1.768$$

$The\ calculated\ value\ of\ \ Z = 1.768 < 2.58\ (Significant\ value\ or\ table\ value\ of\ Z)$

$Conclusion$: *It is not significant at 1% level of significance, hence there is no reason to reject* H_0 ($\therefore$ *we accept* H_0) *i.e., we conclude that the ambulance service claim is accepted.*

Example2: It is claimed that an automobile is driven on the average more than 20,000 kilometres per year. To test this claim, a random sample of 100 automobile owners is asked to keep a record of the kilometres they travel. Would you agree with this claim if the random sample showed an average of 23,500 kilometres and a standard deviation of 3900 kilometres?

Solution: Given that

$specified\ (True)\ Mean\ \mu = 20{,}000,\quad standard\ \ deviation\ \ \sigma = 3900,$

$$sample\,size\ \ n = 100,\ \ sample\,mean\ \bar{x} = 23{,}500$$

we have to test

$Null\ Hypothesis \qquad H_0 : \mu = 20{,}000$

$Alternate\ \ Hypothesis \quad H_1 : \mu > 20{,}000\ (one-tailed\ \ test)$

$Level\ \ of\ \ significance \quad \alpha = 0.05$

$$Test\ \ statistic\ Z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{23500-20000}{3900/\sqrt{100}} = \frac{3500}{390} = 8.974$$

$The\ calculated\ value\ of\ \ Z = 8.974 > 1.645\ (Significant\ value\ or\ table\ value\ of\ Z)$

$Conclusion$: *It is highly significant at 5% level of significance, hence we reject* H_0 (*accept* H_1) *i.e., we conclude that the automobile is driven on the average more than 20,000 kilometers per year.*

2. Z - test for DIFFERENCE OF MEANS

TEST OF SIGNIFICANCE FOR DIFFERENCE OF MEANS

Let $\overline{x_1}$ be the mean of a random sample of size n_1 drawn from a population with mean μ_1 and variance σ_1^2 and let $\overline{x_2}$ be the mean of an independent random sample of size n_2 from another population with mean μ_2 and variance σ_2^2, then for testing the significant difference between the two means we consider the test $H_0 : \mu_1 = \mu_2$

Then the test statistic corresponding to the difference of sample means $(\overline{x_1} - \overline{x_2})$ is given by

$$Z = \frac{(\overline{x_1} - \overline{x_2}) - E(\overline{x_1} - \overline{x_2})}{S.E(\overline{x_1} - \overline{x_2})} = \frac{(\overline{x_1} - \overline{x_2}) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$\Rightarrow \quad Z = \frac{(\overline{x_1} - \overline{x_2})}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \ (under\ H_0)$$

Remark1: If $\sigma_1^2 = \sigma_2^2 = \sigma^2$ i.e., if the samples have been drawn from the populations with common S.D. σ then under $H_0 : \mu_1 = \mu_2$ the test statistic $Z = \frac{(\overline{x_1} - \overline{x_2})}{\sqrt{\sigma^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$

Remark2: If $\sigma_1^2 \neq \sigma_2^2$ and σ_1^2, σ_2^2 are not known then they are estimated from the sample values. If samples are large these estimates are given by $\sigma_1^2 = s_1^2$ *and* $\sigma_2^2 = s_2^2$ then under $H_0 : \mu_1 = \mu_2$ the test statistic is given by $Z = \frac{(\overline{x_1} - \overline{x_2})}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$

<u>Conclusion:</u> If the calculated value of $|Z|$ is greater than the table value of Z at the level of significance α then we reject H_0 otherwise we accept H_0.

Example1: The means of two single large samples of 1000 and 2000 members are 67·5 inches and 68·0 inches respectively. Can the samples be regarded as drawn from the same population of standard deviation 2·5 inches? (Test at 5% level 0f significance).

Solution: we are given $n_1 = 1000; n_2 = 2000$ *and* $\overline{x_1} = 67.5$; $\overline{x_2} = 68.0$ *and* $\sigma = 2.5$

Null Hypothesis $H_0 : \mu_1 = \mu_2$ *i.e., the samples have been drawn from the same population of standard deviation* $\sigma = 2.5$

Alternate Hypothesis $H_1 : \mu_1 \neq \mu_2$ *(Two-tailed test)*

Level of significance $\alpha = 0.05$

$$Test\ stastisc\ under\ H_0\ is\ Z = \frac{(\overline{x_1} - \overline{x_2})}{\sqrt{\sigma^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{67.5 - 68.0}{\sqrt{(2.5)^2\left[\frac{1}{1000} + \frac{1}{2000}\right]}} = \frac{-0.5}{0.09675} = -5.168$$

Since I Z I > 3, the value is highly significant and, we reject the null hypothesis. Hence we conclude that samples are certainly not drawn from the same population with standard deviation 2·5.

Example2: The mean height 0f 50 male students who showed above average participation in college athletics was 68·2 inches with a standard deviation 0f 2·5 inches; while 50 male students who showed no interest in such participation had a mean height 0f 67·5 inches with a standard deviation 0f 2·8 inches. Test the hypothesis that male students who participate in college athletics are taller than other male students.

Solution: we are given $n_1 = 50,\ n_2 = 50\ and\ \overline{x_1} = 68.2,\ s_1 = 2.5;\ \overline{x_2} = 67.5, s_2 = 2.8$

Null Hypothesis $H_0 : \mu_1 = \mu_2$ *i.e., There is no significant difference in mean heights*

Alternate Hypothesis $H_1 : \mu_1 > \mu_2$ *(Right – tailed test) The mean height of students who participate in athletics are taller than others*

Level of significance $\alpha = 0.05$

$$Test\ stastisc\ under\ H_0\ is\ \ Z = \frac{\left(\overline{x_1} - \overline{x_2}\right)}{\sqrt{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)}} = \frac{68.2 - 67.5}{\sqrt{\left[\frac{6.25}{50} + \frac{7.84}{50}\right]}} = \frac{0.7}{\sqrt{0.2818}} = \frac{0.7}{0.5308} = 1.3187$$

Since Z=1.3187 < 1.645, the value is not significant, no reason to reject the null hypothesis. Hence we conclude that the mean height of male students who participate in college athletics are not taller than other male students who showed no interest in athletics.

Example3: Two types of new cars produced in U.S.A. are tested for petrol mileage, a sample from type1 consisting of 42 cars averaged 15 Kmpl while the sample from type2 consisting of 80 cars averaged 11.5 Kmpl with population variances as 2.0 and 1.5 respectively. Test whether there is any significant difference in the petrol consumption of these two types of cars at 0.01 level of significance.

Solution: we are given $n_1 = 42,\ n_2 = 80\ and\ \overline{x_1} = 15,\ \sigma_1^{\ 2} = 2.0;\ \overline{x_2} = 11.5, \sigma_2^{\ 2} = 1.5$

Null Hypothesis $H_0 : \mu_1 = \mu_2$ *(There is no difference in mileage of two types of cars)*

Alternate Hypothesis $H_1 : \mu_1 \neq \mu_2$ *(Two – tailed test)*

Level of significance $\alpha = 0.01$

$$Test\ stastisc\ under\ H_0\ is\ \ Z = \frac{\left(\overline{x_1} - \overline{x_2}\right)}{\sqrt{\left(\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)}} = \frac{15 - 11.5}{\sqrt{\left[\frac{2.0}{42} + \frac{1.5}{80}\right]}} = \frac{3.5}{\sqrt{0.0664}} = \frac{3.5}{0.2576} = 13.586$$

Since Z=13.586 > 3, the value is highly significant, and we reject the null hypothesis H_0 .

We accept H_1 , Hence we conclude that there is a significant difference in petrol consumption of two types of cars.

Example4: The mean yield of samples of two sets of plots and their variability are as given below. Examine whether the difference in the mean yields of the two sets of plots is significant.

	Set-1	Set-2
No. of plots	40	60
Mean yield per plot	1258	1243
Standard deviation	34	28

Solution: we are given $n_1 = 40,\ n_2 = 60\ and\ \overline{x_1} = 1258,\ s_1 = 34;\ \overline{x_2} = 1243, s_2 = 28$

$Null\ Hypothesis\quad H_0 : \mu_1 = \mu_2$ i.e., There is no significant difference in mean yields

$Alternate\ Hypothesis\quad H_1 : \mu_1 \neq \mu_2$ (Two – tailed test) There is significant difference between the mean yields

$Level\ of\ significance\ \alpha = 0.05$

$$Test\ stastisc\ under\ H_0,\ Z = \frac{(\overline{x_1} - \overline{x_2})}{\sqrt{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)}} = \frac{1258 - 1243}{\sqrt{\left[\frac{1156}{40} + \frac{784}{60}\right]}} = \frac{15}{\sqrt{41.9667}} = \frac{0.7}{0.5308} = 2.315$$

Since calculated value Z=2.315 > 1.96 (significant value of Z at 5% level), the value is significant, therefore we reject the null hypothesis H_0.

Hence we accept H_1, and we conclude that there is a significant difference in the yields of two sets of plots.

3. Z - test for SINGLE PROPORTION

TEST OF SIGNIFICANCE FOR SINGLE PROPORTION

Let 'x' is the number of number of successes corresponding to a particular attribute in a sample of 'n' observations, then the proportion of successes in the given sample p=x/n (say).

Then for testing the statistical hypothesis that the significance difference between the sample proportion and a specified Population proportion 'P_0' (or testing the hypothesis that the given sample has been drawn from a specified population of proportion 'P_0') as $H_0 : P = P_0$

The test statistic is given by $Z = \frac{p - P}{\sqrt{\frac{PQ}{n}}}$ (under H_0) where Q=1-P

Conclusion: If the calculated value of $|Z|$ is greater than the table value of Z at the level of significance α then we reject H_0 otherwise we accept H_0.

Example1: A salesclerk in the departmental store claims that 60% of the shoppers entering the store leave without making a purchase. A random sample of 50 shoppers showed that 35 of them left without buying anything. Are these sample results consistent with the claim of the salesclerk? Use 0.05 level of significance.

Solution: Given that sample size n=50, No. of shoppers left without buying x=35

Sample proportion of shoppers left without buying $p = \frac{x}{n} = \frac{35}{50} = 0.7$

$we\ \ have\ \ to\ \ test$

$Null\ \ Hypothesis\ H_0 : P = 0.6\ \ (Q = 1 - P = 0.4)\ \ (60\%\ of\ \ shoppers\ leave\ \ without\ \ buying)$

$Alternate\ Hypothesis\ H_1 : P \neq 0.6\,(Two - tailed\ \ test)$

$Level\ of\ \ significance\ \alpha = 0.05$

$$Test\ \ Statistic\ \ Z = \frac{p-P}{\sqrt{\frac{PQ}{n}}} = \frac{0.7-0.6}{\sqrt{\frac{0.6\times 0.4}{50}}} = \frac{0.1}{\sqrt{0.0048}} = 1.443$$

Since Z=1.443 < 1.96, the value is not significant, there is no reason to reject the null hypothesis H_0. Hence we accept H_0 and can conclude that the sales clerk's claim of 60% of shoppers leave without buying is acceptable.

Example2: A commonly prescribed drug for relieving nervous tension is believed to be only 60% effective. Experimental results with a new drug administered to a random sample of 100 adults who were suffering from nervous tension show that 70 people got relief. Is this sufficient evidence to conclude that the new drug is superior to the one commonly prescribed? Use a 0.05 level of significance.

Solution: Given that sample size n=100, No. of people got relief in sample x=70

Sample proportion of people got relief $p = \frac{x}{n} = \frac{70}{100} = 0.7$

$we\ \ have\ \ to\ \ test$

$Null\ \ Hypothesis\ H_0 : P = 0.6\ \ (Q = 1 - P = 0.4)\ \ (New\, drug\ is\ also\ \ 60\%\ effective\ as\ common\ \ drug)$

$Alternate\ Hypothesis\ H_1 : P > 0.6\,(One - tailed\ \ test)$

$(New\, drug\ is\ \sup erior\ \ when\ \ compared\ \ to\ \ common\ \ drug)$

$Level\ of\ \ significance\ \alpha = 0.05$

$$Test\ \ Statistic\ \ Z = \frac{p-P}{\sqrt{\frac{PQ}{n}}} = \frac{0.7-0.6}{\sqrt{\frac{0.6\times 0.4}{100}}} = \frac{0.1}{\sqrt{0.0024}} = 2.04$$

Since Z=2.04 > 1.645, the value is significant, so we reject the null hypothesis H_0.

Hence, we accept H_1, and can conclude that the new drug is superior to the common drug to get relief from nervous tension.

Example3: In a sample of 1,000 people in Maharashtra, 540 are rice eaters and the rest are wheat eaters. Can we assume that both rice and wheat are equally popular in this State at 1% level of significance?

Solution: Given that sample size n=1,000; No. of rice eaters in the sample x=540

Sample proportion of rice eaters $p = \frac{x}{n} = \frac{540}{1000} = 0.54$

$we\ \ have\ \ to\ \ test$

$Null\ \ \ Hypothesis\ H_0 : P = 0.5\ \ \ (Q = 1 - P = 0.5)\ \ (The\ \ proportion\ of\ \ rice\ Eaters\ are\ 50\%)$

$(Both\ rice\ and\ \ wheat\ are\ eqully\ \ popular)$

$Alternate\ Hypothesis\ H_1 : P \neq 0.5 (Two - tailed\ \ test)\ (Rice\ and\ \ wheat\ are\ not\ \ eqully\ \ popular)$

$Level\ \ of\ \ significance\ \alpha = 0.01$

$$Test\ \ \ Statistic\ \ Z = \frac{p - P}{\sqrt{\frac{PQ}{n}}} = \frac{0.54 - 0.5}{\sqrt{\frac{0.5 \times 0.5}{1000}}} = \frac{0.04}{\sqrt{0.00025}} = 2.5298$$

Since Z=2.5298<2.58, the value is not significant, no reason to reject the null hypothesis H_0 . Hence we accept H_0, and can conclude that both rice and wheat are equally popular in Maharashtra.

Example4: A researcher claims that at least 10% of all football helmets have manufacturing flaws that could potentially cause injury to the wearer. A sample of 200 helmets revealed that 16 helmets contained such defects. Does this finding support the researcher's claim? (Use 0.01 level of significance.)

Solution: Given that sample size n=200; No. of defective helmets in the sample x=16

Sample proportion of defective helmets $p = \frac{x}{n} = \frac{16}{200} = 0.08$

$we\ \ have\ \ to\ \ test$

$Null\ \ \ Hypothesis\ H_0 : P = 0.1\ \ \ (Q = 1 - P = 0.9)\ \ (The\ \ proportion\ of\ \ defectives\ 10\%)$

$Alternate\ Hypothesis\ H_1 : P \geq 0.1 (One - tailed\ \ test)$

$(The\ \ proportion\ of\ \ defectives\ are\ atleast\ 10\%)$

$Level\ \ of\ \ significance\ \alpha = 0.01$

$$Test\ \ \ Statistic\ \ Z = \frac{p - P}{\sqrt{\frac{PQ}{n}}} = \left|\frac{0.08 - 0.1}{\sqrt{\frac{0.1 \times 0.9}{200}}}\right| = \left|\frac{0.02}{0.0212}\right| = 0.9434$$

Since Z=0.9434 < 2.33, the value is not significant, There no reason to reject the null hypothesis H_0 .

Hence we accept H_0, and we can conclude that the defectives are not more than 10%.

4. Z - test for DIFFERENCE OF PROPORTIONS

TEST OF SIGNIFICANCE FOR DIFFERENCE OF PROPORTIONS

Suppose we want to compare two distinct populations with respect to the prevalence of a certain attribute, say A, among their members. Let X_1 *and* X_2 be the number of persons possessing the given attribute 'A' in random samples of sizes n_1 *and* n_2 from the two populations respectively. Then sample proportions are given by $p_1 = \frac{X_1}{n_1}$ *and* $p_1 = \frac{X_1}{n_1}$

Then for testing the significance difference between the proportions we consider $H_0 : P_1 = P_2$ where P_1 *and* P_2 population proportions are

Then the test statistic corresponding to the difference of sample proportions $(p_1 - p_2)$ is given by

$$Z = \frac{(p_1 - p_2) - E(p_1 - p_2)}{S.E(p_1 - p_2)} = \frac{(p_1 - p_2) - (P_1 - P_2)}{\sqrt{\frac{P_1Q_1}{n_1} + \frac{P_2Q_2}{n_2}}}$$

$$\Rightarrow \quad Z = \frac{(p_1 - p_2)}{\sqrt{PQ\left(\frac{1_1}{n_1} + \frac{1}{n_2}\right)}} \quad (under\ H_0 : P_1 = P_2)$$

An unbiased estimate of the population proportion P based on the samples is

$$P = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} = \frac{x_1 + x_2}{n_1 + n_2} \qquad then\ Q = 1 - P$$

- Suppose the population proportions P_1 and P_2 are given to be distinctly different, i.e., $P_1 \neq P_2$ and we want to test if the difference $(P_1 - P_2)$ in population proportions is likely to be hidden in simple samples of sizes n_1 and n_2 from the two populations respectively. Then the test statistic is given by

$$Z = \frac{(P_1 - P_2)}{\sqrt{\frac{P_1Q_1}{n_1} + \frac{P_2Q_2}{n_2}}} \quad \text{(Here the sample proportions are not given)}$$

<u>Conclusion:</u> If the calculated value of $|Z|$ is greater than the table value of Z at the level of significance α then we reject H_0 otherwise we accept H_0.

Example1. In a study to estimate the proportion of residents in a certain city and its suburbs who favour the construction of a nuclear power plant, it is found that 63 of 100 urban residents favour the construction while only 59 of 125 suburban residents are in favour. Is there a significant difference between the proportion of urban and suburban residents who favour construction of the nuclear plant? (Use 0.05 level of significance)

Solution: It is given that $n_1 = 100; X_1 = 63\ and\ n_2 = 125; X_2 = 59$ then

The sample proportion of residents who favour the construction of nuclear plant

$$p_1 = \frac{X_1}{n_1} = \frac{63}{100} = 0.63 \quad and \quad p_2 = \frac{X_2}{n_2} = \frac{59}{125} = 0.472$$

$$Null\ Hypothesis\quad H_0: P_1 = P_2\quad (proportion\ of\ residents\ in\ city\ and\ suburban\ are\ same)$$

$$Alternate\ Hypothesis\ \ H_1: P_1 \neq P_2\ \ (Two-tailed\ test)$$

$$Level\ of\ significance\ \alpha = 0.05$$

$$Test\ statistic\ Z = \frac{(p_1 - p_2)}{\sqrt{PQ\left(\frac{1_1}{n_1} + \frac{1}{n_2}\right)}}\quad (under\ H_0: P_1 = P_2)$$

$$An\ unbiased\ estimate\ of\ the\ population\ proportion\ P\ based\ on\ the\ samples\ is$$

$$P = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{63 + 59}{100 + 125} = \frac{122}{225} = 0.542$$

$$then\ Q = 1 - P = 1 - 0.542 = 0.458$$

$$Now\ \ Z = \frac{(p_1 - p_2)}{\sqrt{PQ\left(\frac{1_1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.63 - 0.472}{\sqrt{0.542 \times 0.458\left[\frac{1}{100} + \frac{1}{125}\right]}} = \frac{0.158}{\sqrt{0.00447}} = 2.363$$

Since Z=2.363 > 1.96, the value is significant, so we reject the null hypothesis H_0 .

Hence we accept H_1, and we can conclude that the proportion of residents in city and suburban are not same in favour of the construction of nuclear plant at 5% level.

Example2: A company is considering two different television advertisements for promotion of a new product. Management believed that the advertisement A is more effective than advertisement B. Two test market areas with virtually identical consumer characteristics are selected; A is used in one area and B in other area. In a random sample of 60 customers who saw A, 18 tried the product. In another random sample of 100 customers who saw B, 22 tried the product. Does this indicate that advertisement A is more effective than advertisement B, if a 5% level of significance is used?

Solution: It is given that $n_1 = 60; X_1 = 18\ and\ n_2 = 100; X_2 = 22$ then

The sample proportion of customers who tried the product by watching A and B are

$$p_1 = \frac{X_1}{n_1} = \frac{18}{60} = 0.3\quad and\quad p_2 = \frac{X_2}{n_2} = \frac{22}{100} = 0.22$$

$$Null\ Hypothesis\quad H_0: P_1 = P_2\quad (The\ advertisements\ A\ and\ B\ are\ eqully\ effectie)$$

$$Alternate\ Hypothesis\ \ H_1: P_1 > P_2\ \ (one-tailed\ test)\ (A\ is\ more\ effectve\ than\ B)$$

$$Level\ of\ significance\ \alpha = 0.05$$

$$Test\ statistic\ Z = \frac{(p_1 - p_2)}{\sqrt{PQ\left(\frac{1_1}{n_1} + \frac{1}{n_2}\right)}}\quad (under\ H_0: P_1 = P_2)$$

$$where\quad P = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} = \frac{X_1 + X_2}{n_1 + n_2} = \frac{18 + 22}{60 + 100} = \frac{40}{160} = 0.25$$

$$then\ Q = 1 - P = 1 - 0.25 = 0.75$$

$$Now\ \ Z = \frac{(p_1 - p_2)}{\sqrt{PQ\left(\frac{1_1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.3 - 0.22}{\sqrt{0.25 \times 0.75\left[\frac{1}{60} + \frac{1}{100}\right]}} = \frac{0.08}{\sqrt{0.005}} = 1.1314$$

Since Z=1.1314 < 1.645, the value is not significant, There no reason to reject the null hypothesis H_0 .

Hence we accept H_0, and we can conclude that the advertisements A and B are equally effective.

Example3: A machine puts out 16 imperfect articles in a sample of 500. After machine is overhauled, it puts out 3 imperfect articles in a batch of 100. Has the machine improved? (Test at 0.01 level of significance)

Solution: Given that $n_1 = 500;\ X_1 = 16 \quad and \quad n_2 = 100;\ X_2 = 3$

Then the proportion of imperfect articles in the samples before and after overhauling the machine $p_1 = \frac{X_1}{n_1} = \frac{16}{500} = 0.032 \quad and \quad p_2 = \frac{X_2}{n_2} = \frac{3}{100} = 0.03$

$$Null\ Hypothesis \quad H_0 : P_1 = P_2 \quad (No\ signficant\ difference\ in\ the\ proportion\ of\ defectives\ articles\ before\ and\ after\ overhauling\ the\ machine)$$

$$Alternate\ Hypothesis \quad H_1 : P_1 > P_2 \quad (one-tailed\ test)\ (\Pr oportion\ of\ imperfect\ articles\ are\ decreased\ after\ overhauled\ the\ machine, \therefore the\ machine\ has\ improved)$$

$$Level\ of\ significance\ \alpha = 0.01$$

$$Test\ statistic\ Z = \frac{(p_1 - p_2)}{\sqrt{PQ\left(\frac{1_1}{n_1} + \frac{1}{n_2}\right)}} \quad (under\ H_0 : P_1 = P_2)$$

$$where \quad P = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} = \frac{X_1 + X_2}{n_1 + n_2} = \frac{16+3}{500+100} = \frac{19}{600} = 0.0317$$

$$then\ Q = 1 - P = 1 - 0.0317 = 0.9683$$

$$Now\ Z = \frac{(p_1 - p_2)}{\sqrt{PQ\left(\frac{1_1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.032 - 0.03}{\sqrt{0.0317 \times 0.9683\left[\frac{1}{500} + \frac{1}{100}\right]}} = \frac{0.002}{\sqrt{0.000368}} = 0.1043$$

Since Z=0.1043 < 2.33, the value is not significant, There no reason to reject the null hypothesis H_0 .

Hence we accept H_0, and we can conclude that there no significant difference in two proportions of imperfect articles before and after overhauled the machine.

So, we can say that the machine has not improved.

Example5: Before an increase in excise duty on tea, 800 persons out of a sample of 1000 persons were found to be consumers of tea. After an increase in excise duty, 800 persons were consumers of tea in a sample of 1200. State whether there is any significant decrease in the consumption of tea after the increase in excise duty? (Use 5% level of significance)

Solution: Given that $n_1 = 1000;\ X_1 = 800 \quad and \quad n_2 = 1200;\ X_2 = 800$

Then, the proportion of consumers of tea in the samples before and after increasing the excise duty on tea $p_1 = \frac{X_1}{n_1} = \frac{800}{1000} = 0.8 \quad and \quad p_2 = \frac{X_2}{n_2} = \frac{800}{1200} = 0.667$

$$Null\ Hypothesis \quad H_0 : P_1 = P_2 \quad (No\ signficant\ difference\ in\ the\ proportion\ of\ consumers\ of\ tea\ before\ and\ after\ increase\ in\ excise\ dury)$$

$$Alternate\ Hypothesis\ H_1 : P_1 > P_2\ (one-tailed\ test)\ (Proportion\ of\ consumers\ of\ tea\ is\ decreased\ after\ increase\ in\ excise\ duty)$$

$$Level\ of\ significance\ \alpha = 0.05$$

$$Test\ statistic\ Z = \frac{(p_1 - p_2)}{\sqrt{PQ\left(\frac{1_1}{n_1} + \frac{1}{n_2}\right)}}\ (under\ H_0 : P_1 = P_2)$$

$$where\quad P = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} = \frac{X_1 + X_2}{n_1 + n_2} = \frac{800 + 800}{1000 + 1200} = \frac{1600}{2200} = 0.7273$$

$$then\ Q = 1 - P = 1 - 0.7273 = 0.2727$$

$$Now\ \ Z = \frac{(p_1 - p_2)}{\sqrt{PQ\left(\frac{1_1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.8 - 0.667}{\sqrt{0.7273 \times 0.2727\left[\frac{1}{1000} + \frac{1}{1200}\right]}} = \frac{0.133}{\sqrt{0.00036}} = 6.9748$$

Since Z=6.9748 > 1.645, the value is highly significant, we reject the null hypothesis H_0 . Hence we accept H_1, and we can conclude that there is a significant decrease in the consumption of tea after increase in excise duty.

PRACTICE QUESTIONS

1. A random sample of 100 recorded deaths in the United States during the past year showed an average life span of 71.8 years. Assuming a population standard deviation of 8.9 years, does this seem to indicate that the: mean life span today is greater than 70 years? Use a 0.05 level of significance.

2. The mean breaking strength of cables supplied by a manufacturer is 1800 with a standard deviation 100. By a new technique in the manufacturing process, it is claimed that the breaking strength of the cables has increased. In order to test this, claim a sample of 50 cables is tested. It is found that the mean breaking strength is 1850. Can we support the claim at 0·01 level of significance?

3. A simple sample of heights of 6,400 Englishmen has a mean of 67·85 Inches and S.D. 2·56 inches, while a simple sample of heights of 1,600 Australians has a mean of 68·55 inches and a S.D. of 2·52 inches. Do the data indicate that Australians are, on the average, taller than Englishmen?

4. At a certain large university, a sociologist speculates that male students spend considerably more money on junk food than do female students. To test the hypothesis, the sociologist randomly selects from the registrars records the names of 200 students. Of these 125 are men and 75are women. The average amount spent on junk food per week by the men is Rs.400/- and standard deviation is Rs100/-. For the women, the sample mean is Rs.450/- and the S.D is Rs.150/-.Test the significant difference between the means at 0.05 level of significance.

5. Two samples of students taken from different universities, the means of their weights and standard deviations are given; make a large sample test to verify the significant difference between the means.

	Mean weight	Standard Deviation of weight	Size of the sample
University A	55	10	400
University B	57	15	100

6. A builder claims that heat pumps are installed in 70% of all homes being constructed today in the city of Richmond, Virginia. Would you agree with this claim if a random survey of new homes in this city shows that 8 out of 15 had heat pumps installed? Use a 0.10 level of significance.

7. A personnel manager claims that 80 per cent of all single women hired for secretarial job get married and quit work within two years after they are hired. Test this hypothesis at 5% level of significance if among 200 such secretaries, 112 got married within two years after they were hired and quit their jobs.

8. In a large consignment of oranges, a random sample of 64 oranges revealed that 14 oranges were bad. Is it reasonable to assume that 20% of the oranges were bad in the consignment?

9. A company has the head office at Kolkata and a branch at Mumbai. The personnel director wanted to know if the workers at the two places would like the introduction of a new plan of work and a survey was conducted for this purpose. Out of a sample of 500 workers at Kolkata 62% favoured the new plan. At Mumbai out of a sample of 400 workers, 41% were against the new plan. Is there any significant difference between the two groups in their attitude towards the new plan at 5% level? 0.917

10. A firm, manufacturing dresses for children, sent out advertisement through mail. Two groups of 1,000 each were contacted: the first group having been contacted in white covers while the second in blue covers. 20% from the first while 28% from the second replied. Do you think that blue envelopes help the sales?

11. If 57 out of 150 patients suffering with certain disease cured by allopathy and 33 out of 100 patients with same disease are cured by homeopathy. Is there any reason to believe that allopathy is better than homeopathy at 1 % level of significance?

SUMMARY

- Hypothesis, Null and Alternative Hypothesis, Type-I and Type-II Errors.
- Steps in the procedure for testing of Hypothesis.
- Z-test for single mean : To test the significant difference between the sample mean $\bar{x}$ and population mean μ the test statistic is $Z = \dfrac{\bar{x}-\mu}{\sigma/\sqrt{n}}$
- Z-test for difference of means : To test the significant difference between the means of two populations the test statistic is $Z = \dfrac{(\bar{x}_1-\bar{x}_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1}+\dfrac{\sigma_2^2}{n_2}}}$
- Z-test for single proportion : To test the significant difference between the sample proportion 'p' and population proportion 'P' the test statistic is $Z = \dfrac{p-P}{\sqrt{\dfrac{PQ}{n}}}$
- Z-test for difference of proportions: To test the significant difference between the proportions of two populations the test statistic is $Z = \dfrac{(p_1-p_2)}{\sqrt{PQ\left(\dfrac{1_1}{n_1}+\dfrac{1}{n_2}\right)}}$

$$$$$$

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UNIT-4

SMALL SAMPLE TESTS (n<30)

1. t - test for SINGLE MEAN

TEST OF SIGNIFICANCE FOR SINGLE MEAN

Let us consider $x_1, x_2, x_3, \ldots. x_n$ is a random sample of size n taken from a normal population and let $\bar{x}$ is the mean of this sample also follows a normal distribution,

Then to test the significance difference between the sample mean $\bar{x}$ and the specified population mean μ_0 (or to test whether the sample has been drawn from a normal population of mean μ_0)

Under the Null Hypothesis:

(i) The sample has been drawn from the population with mean μ or

(ii)There is no significant difference between the sample mean $\bar{x}$ and population mean μ

That is for testing $H_0 : \mu = \mu_0$ (Null Hypothesis)

The test statistic is given by $t = \dfrac{\bar{x} - \mu_0}{S/\sqrt{n}}$ or $t = \dfrac{\bar{x} - \mu}{S/\sqrt{n}}$ (under H_0)

Where $S = \sqrt{\dfrac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2} = \sqrt{\dfrac{1}{n-1}\left(\sum x_i^2 - \dfrac{\left(\sum x_i\right)^2}{n}\right)}$

The statistic 't' follows a student's t-distribution with n-1 degrees of freedom.

Conclusion: If the calculated value of $|t|$ is greater than the table value of 't' at the level of significance α then we reject H_0 otherwise we accept H_0.

Example1: The average breaking strength of steel rods is specified to be 18·5 thousand pounds. To test this, a sample of 14 rods was tested. The mean and standard deviations obtained were 17·85 and 1.955 thousand pounds respectively. Is the result of the experiment significant?

Solution: Given that *sample size* $n = 14$ *samplemean* $\bar{x} = 17.85$
standard deviation $s = 1.955$, *Specified (Expected) Mean* $\mu = 18.5$

we have to test

Null Hypothesis $H_0 : \mu = 18.5$ (*No change in average breaking strength*)

Alternate Hypothesis $H_1 : \mu \neq 18.5$ (*Two-tailed test*)(*some change in average breaking strength*)

Level of significance $\alpha = 0.05$

$$\text{Test statistic } t = \left|\frac{\bar{x} - \mu}{s/\sqrt{n}}\right| = \left|\frac{17.85 - 18.5}{1.955/\sqrt{14}}\right| = \left|\frac{-0.65}{0.5225}\right| = 1.244$$

The significant value of t at 0.05 *level with* $n-1 = 13$ *d.f for two-tailed test is* 2.16

The calculated value of $t = 1.244 < 2.16$ (*Significant value or table value*)

Conclusion : It is not significant at 5% level of significance, hence we cannot reject H_0

i.e.,we accept H_0 *and we Can conclude that there is no change in average breaking strength*

Example2: The mean weekly sales of soap bars in departmental stores were 146·3 bars per store. After an advertising campaign the mean weekly sales in 22 stores for a typical week increased to 153·7 and showed a standard deviation of 17·2. Was the advertising campaign successful? (Test at 0.05 level)

Solution: Given that

$sample\ size\ n = 22\ \ sample\ mean\ \bar{x} = 153.7$

$standard\ \ deviation\ \ s = 17.2, \qquad Comparative(Expected)\ Mean\ \ \mu = 146.3$

$we\ \ have\ \ to\ \ test$

$Null\ \ Hypothesis \qquad H_0 : \mu = 146.3 \qquad (No\ change\ \ in\ \ average\ \ sales\ of\ soap\ \ bars)$

$Alternate\ \ Hypothesis\ \ H_1 : \mu > 146.3\ (One-tailed\ \ test)\ \ (Average\ \ sales\ \ has\ \ been\ \ \ increased)$

$Level\ \ of\ \ significance \quad \alpha = 0.05$

$$Test\ \ \ statistic\ \ t = \left|\frac{\bar{x}-\mu}{s/\sqrt{n}}\right| = \left|\frac{153.7-146.3}{17.2/\sqrt{22}}\right| = \left|\frac{7.4}{3.667}\right| = 2.0179$$

$The\ significant\ value\ of\ t\ \ at\ \ 0.05\ \ level\ with\ \ n-1 = 21\ d.f\ \ for\ one-tailed\ \ test\ \ is\ 1.721$

$The\ \ calculated\ \ value\ \ of\ \ t = 2.0179 > 1.721\ (Significant\ value\ \ or\ \ table\ \ value)$

$Conclusion : It\ \ is\ \ significant\ \ at\ 5\%\ \ level\ \ of\ \ significance,\ \ hence\ \ we\ \ reject\ \ H_0,$

$i.e., we\ \ accept\ \ H_1\ \ and\ \ we\ \ Can\ \ conclude\ \ that\ there\ is\ \ significant\ \ change(increase)\ \ in\ \ average\ \ sales$

$Hence\ \ we\ can\ say\ \ that\ \ the\ \ advertising\ \ campaign\ \ was\ \ successful.$

Example3: Ten individuals are chosen at random from a normal population and their heights are found to be 63, 64, 66, 67, 68, 69, 70, 70, 71, 72 inches. Test if the sample belongs to the population whose mean height is 66", use 0.01 level of significance.

Solution: From the given data

$sample\ \ size\ \ n = 10\ \ sample\ \ mean\ \ \bar{x} = 68\ (calculated)$

$standard\ \ deviation\ \ \ s = 2.9814\ (calculated),\ \ \ Comparative(Expected)\ Mean\ \ \mu = 66$

values(x_i)	$\sum(x_i-\bar{x})^2$
63	25
64	16
66	4
67	1
68	0
69	1
70	4
70	4
71	9
72	16
680	**80**

$$\text{Sample Mean } \bar{x} = \frac{\sum x_i}{n} = \frac{680}{10} = 68$$

$$S = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2} = \sqrt{\frac{1}{n-1}\left(\sum x_i^2 - \frac{\left(\sum x_i\right)^2}{n}\right)}$$

$$= \sqrt{\frac{1}{9}(80)} = 2.9814$$

we have to test

Null Hypothesis $H_0 : \mu = 66$ (*No change in average height*)

Alternate Hypothesis $H_1 : \mu \neq 66$ (*two-tailed test*) (*A significant change in average height*)

Level of significance $\alpha = 0.01$

$$Test\ \ statistic\ \ t = \left|\frac{\bar{x}-\mu}{s/\sqrt{n}}\right| = \left|\frac{68-66}{2.9814/\sqrt{10}}\right| = \left|\frac{2}{0.9428}\right| = 2.1213$$

The significant value of t at 0.01 *level with* $n-1=9\,d.f$ *for two-tailed test is* 3.25

The calculated value of $t = 2.1213 < 3.25$ (*Significant value or table value*)

Conclusion : It is not significant at 1% *level of significance, hence we cannot reject* H_0,

i.e.,we accept H_0 *and we Can conclude that the average height of population is* 66 *inches*.

2. t - test for DIFFERENCE OF MEANS

TEST OF SIGNIFICANCE FOR DIFFERENCE OF MEANS

Suppose we want to test if (i) Two independent samples $x_i\,(i=1,2,....n_1)$ and $x_j\,(j=1,2,....n_2)$ have been drawn from the populations with same means $(\mu_1 = \mu_2)$ or (b) The two sample means $\bar{x}_1$ *and* $\bar{x}_2$ differ significantly or not.

Under the null hypothesis, that (i) samples have been drawn from the populations with the same means, i.e., $(H_0 : \mu_1 = \mu_2)$ or (ii) the sample means $\bar{x}_1$ *and* $\bar{x}_2$ do not differ significantly, then the test statistic is given by....

$$\Rightarrow \qquad t = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{S^2\left[\frac{1}{n_1}+\frac{1}{n_2}\right]}} \qquad (under\ \ H_0)$$

Which follows a Student's t-distribution with $(n_1 + n_2 - 2)$ degrees of freedom

$$where\ S^2 = \frac{1}{n_1+n_2-2}\left[\sum_{i=1}^{n_1}(x_i-\bar{x}_1)^2 + \sum_{j=1}^{n_2}(x_j-\bar{x}_2)^2\right]$$

$$= \frac{1}{n_1+n_2-2}\left[n_1 s_1^2 + n_2 s_2^2\right]$$

Conclusion: If the calculated value of $|t|$ is greater than the table value of t at the level of significance α then we reject H_0 otherwise we accept H_0.

Example1: The melting points of two alloys used in formulating solder were investigated by melting 21 samples of each material. The sample mean and standard deviation for alloy 1 was $\bar{x}_1 = 420^0 F$ *and* $s_1 = 4^0 F$ while for alloy 2 they were $\bar{x}_2 = 426^0 F$ *and* $s_2 = 3^0 F$. Do the sample data support the claim that both alloys have the same melting point? Use α= 0.05 and assume that both populations are normally distributed.

Solution: Given that $n_1 = 21$ $\overline{x_1} = 420,\ s_1 = 4$ and $n_2 = 21,\ \overline{x_2} = 426,\ s_2 = 3$

$we\ \ have\ \ to\ \ test$

$Null\ Hypothesis \quad H_0 : \mu_1 = \mu_2 \quad (No\ significant\ difference\ in\ average\ melting\ points)$

$Alternate\ Hypothesis\ \ H_1 : \mu_1 \neq \mu_2 \quad (Two-tailed\ test)(A\ significant\ change\ in\ melting\ points)$

$Level\ of\ significance \quad \alpha = 0.05$

$$test\ statistic\ \ t = \frac{(\overline{x_1} - \overline{x_2})}{\sqrt{S^2\left[\frac{1}{n_1} + \frac{1}{n_2}\right]}} \quad (under\ H_0)$$

$$where\ S^2 = \frac{1}{n_1 + n_2 - 2}\left[\sum_{i=1}^{n_1}(x_i - \overline{x_1})^2 + \sum_{j=1}^{n_2}(x_j - \overline{x_2})^2\right] = \frac{1}{n_1 + n_2 - 2}\left[n_1 s_1^2 + n_2 s_2^2\right]$$

$$S^2 = \frac{1}{n_1 + n_2 - 2}\left[n_1 s_1^2 + n_2 s_2^2\right] = \frac{1}{21 + 21 - 2}\left[21(4)^2 + 21(3)^2\right] = \frac{1}{40}[336 + 189]$$

$$= \frac{525}{40} = 13.125$$

$$then\ \ t = \left|\frac{(\overline{x_1} - \overline{x_2})}{\sqrt{S^2\left[\frac{1}{n_1} + \frac{1}{n_2}\right]}}\right| = \left|\frac{(420 - 426)}{\sqrt{13.125\left[\frac{1}{21} + \frac{1}{21}\right]}}\right| = \frac{6}{1.118} = 5.3667$$

The significant (table) value of 't' at 0.05 level with $n_1 + n_2 - 2 = 40$ degrees of freedom for a two-tailed test is 2.021

Since calculated value of t=5.3667 > 2.021, t is highly significant.

Conclusion: Therefore, we reject the Null Hypothesis H_0, hence we accept H_1 i.e., we conclude that the two alloys have not the same melting point.

Example2: Samples of two types of electric light bulbs were tested for length of life and following data were obtained.

	Type -1	Type -11
Sample number	n1=8	n2=7
Sample means	x̅1 =1,234 hrs	x̅2 =1,036 hrs
Sample s.d's	s1 =36 hrs	s2 =40 hrs.

Is the difference in the means sufficient to warrant that type-I is superior to type-II regarding length of life?

Solution: Given that $n_1 = 8$ $\overline{x_1} = 1234,\ s_1 = 36$ and $n_2 = 7,\ \overline{x_2} = 1036,\ s_2 = 40$

$we\ \ have\ \ to\ \ test$

$Null\ Hypothesis\qquad H_0 : \mu_1 = \mu_2\quad (No\ significant\ difference\ in\ average\ life\ times)$

$Alternate\ \ Hypothesis\ \ H_1 : \mu_1 > \mu_2\ \ (One-tailed\ test)(Type-I\ is\ superior\ to\ type-II\ bulbs)$

$Level\ of\ \ significance\quad \alpha = 0.05$

$$test\ statistic\ \ t = \frac{(\overline{x_1} - \overline{x_2})}{\sqrt{S^2\left[\frac{1}{n_1} + \frac{1}{n_2}\right]}}\quad (under\ H_0)$$

$$where\ S^2 = \frac{1}{n_1 + n_2 - 2}\left[\sum_{i=1}^{n_1}(x_i - \overline{x_1})^2 + \sum_{j=1}^{n_2}(x_j - \overline{x_2})^2\right] = \frac{1}{n_1 + n_2 - 2}\left[n_1 s_1^2 + n_2 s_2^2\right]$$

$$S^2 = \frac{1}{n_1 + n_2 - 2}\left[n_1 s_1^2 + n_2 s_2^2\right] = \frac{1}{8 + 7 - 2}\left[8(36)^2 + 7(40)^2\right] = \frac{21568}{13} = 1659.0769$$

$$then\ \ t = \left|\frac{(\overline{x_1} - \overline{x_2})}{\sqrt{S^2\left[\frac{1}{n_1} + \frac{1}{n_2}\right]}}\right| = \left|\frac{(1234 - 1036)}{\sqrt{1659.0769\left[\frac{1}{8} + \frac{1}{7}\right]}}\right| = \frac{198}{21.081} = 9.39$$

The significant (table) value of 't' at 0.05 level with $n_1 + n_2 - 2 = 13$ degrees of freedom for a one-tailed test is **1.771**

Since calculated value of t=9.39 > 1.771, t is highly significant.

Conclusion: Therefore we reject the Null Hypothesis H_0, hence we accept H_1 i.e., we conclude that the type-I bulbs are superior to type-II bulbs.

Example3: Two independent groups of 10 children were tested to find how many digits they could repeat from memory after hearing them. The results are as follows:

Group A:	8	6	5	6	6	7	7	4	5	6
Group B:	10	5	7	6	6	9	7	6	7	7

Is the difference between the mean scores of the two groups significant? Use α=0.01

Solution: Given that $n_1 = 10,\ \ n_2 = 10$

$we\ \ have\ \ to\ \ test$

$Null\ Hypothesis\qquad H_0 : \mu_1 = \mu_2\quad (No\ significant\ difference\ between\ the\ mean\ scores)$

$Alternate\ \ Hypothesis\ \ H_1 : \mu_1 \neq \mu_2\ \ (Two-tailed\ test)(Significant\ difference\ between\ the\ mean\ scores)$

$Level\ of\ \ significance\quad \alpha = 0.05$

$$test\ statistic\ \ t = \frac{(\overline{x_1} - \overline{x_2})}{\sqrt{S^2\left[\frac{1}{n_1} + \frac{1}{n_2}\right]}}\quad (under\ H_0)$$

$$where\ S^2 = \frac{1}{n_1 + n_2 - 2}\left[\sum_{i=1}^{n_1}(x_i - \overline{x_1})^2 + \sum_{j=1}^{n_2}(x_j - \overline{x_2})^2\right]$$

x_i	$\sum_{i=1}^{n_1}(x_i-\overline{x_1})^2$	x_j	$\sum_{j=1}^{n_2}(x_j-\overline{x_2})^2$
8	4	10	9
6	0	5	4
5	1	7	0
6	0	6	1
6	0	6	1
7	1	9	4
7	1	7	0
4	4	6	1
5	1	7	0
6	0	7	0
60	**12**	**70**	**20**

From the table we have

$$\overline{x_1}=\frac{60}{10}=6, \qquad \sum_{i=1}^{n_1}(x_i-\overline{x_1})^2=12$$

$$\overline{x_2}=\frac{70}{10}=7, \qquad \sum_{j=1}^{n_2}(x_j-\overline{x_2})^2=20$$

$$S^2=\frac{1}{n_1+n_2-2}\left[\sum_{i=1}^{n_1}(x_i-\overline{x_1})^2+\sum_{j=1}^{n_2}(x_j-\overline{x_2})^2\right]=\frac{1}{10+10-2}[12+20]=\frac{32}{18}=1.7778$$

$$then \quad t=\left|\frac{(\overline{x_1}-\overline{x_2})}{\sqrt{S^2\left[\frac{1}{n_1}+\frac{1}{n_2}\right]}}\right|=\left|\frac{(6-7)}{\sqrt{1.7778\left[\frac{1}{10}+\frac{1}{10}\right]}}\right|=\frac{1}{0.5963}=1.677$$

The significant (table) value of 't' at 0.01 level with $n_1+n_2-2=18$ degrees of freedom for a Two-tailed test is **2.552**

Since calculated value of t=1.4907 < 2.552, t is not significant.

Conclusion: Therefore, we accept the Null Hypothesis H_0, hence we conclude that there is no significant difference between the mean scores of two groups of students.

3. Paired t - test for DIFFERENCE OF MEANS

TEST OF SIGNIFICANCE FOR DIFFERENCE OF MEANS FOR DEPENDENT SAMPLES

Let us consider the case when (i) the sample sizes are equal. i.e., $n_1=n_2=n$ (say) and (ii) the two samples are not independent but the sample observations are paired together. i.e., the pair of observations $(x_i, y_i)\, for\, (i=1,2,......n)$ corresponds to the same (i'th) sample unit. The problem is to test if the sample means differ significantly or not.

Then the test statistic is given by

$$t=\frac{\overline{d}}{S/\sqrt{n}} \quad which\ follows\ a\ student's\ t-distribution\ with\ (n-1)\ degrees\ of\ freedom$$

$$where\ \overline{d}=\frac{\sum d_i}{n} \quad (\because d_i=x_i-y_i) \quad and \quad S=\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(d_i-\overline{d})^2}=\sqrt{\frac{1}{n-1}\left(\sum_{i=1}^{n}d_i^2-\frac{(\sum d_i)^2}{n}\right)}$$

Conclusion: If the calculated value of $|t|$ is greater than the table value of 't' at the level of significance α then we reject H_0 otherwise we accept H_0.

Example1: Eleven schoolboys were given a test in Statistics. They were given a month's coaching, and a second test was held at the end of it. Do the marks give evidence that the students have benefited by the extra coaching? (Test at 0.05 level)

BOYS	1	2	3	4	5	6	7	8	9	10	11
Marks in 1st test(x)	23	20	19	21	18	20	18	17	23	16	19
Marks in 2nd test(y)	24	19	22	18	20	22	20	20	23	20	18

Solution: Given that n=11

Null Hypothesis $H_0 : \mu_1 = \mu_2$ (*No significant difference between marks in two tests*)

Alternate Hypothesis $H_1 : \mu_1 < \mu_2$ (*One − tailed test*)(*Students benefited by extra coaching*)

Level of significance $\alpha = 0.05$

BOYS	Marks in 1st test(x_i)	Marks in 2nd test(y_i)	$d_i=x_i-y_i$	d_i^2
1	23	24	-1	1
2	20	19	1	1
3	19	22	-3	9
4	21	18	3	9
5	18	20	-2	4
6	20	22	-2	4
7	18	20	-2	4
8	17	20	-3	9
9	23	23	0	0
10	16	20	-4	16
11	19	18	1	1
		sum	**-12**	**58**

$$\bar{d} = \frac{\sum d_i}{n} = \frac{-12}{11} = -1.091$$

$$S = \sqrt{\frac{1}{n-1}\left(\sum_{i=1}^{n} d_i^2 - \frac{\left(\sum d_i\right)^2}{n}\right)}$$

$$= \sqrt{\frac{1}{10}\left[58 - \frac{(-12)^2}{11}\right]} = \sqrt{4.491} = 2.1192$$

Test statistic

$$t = \left|\frac{\bar{d}}{S/\sqrt{n}}\right| = \left|\frac{-1.091}{2.1192/\sqrt{11}}\right| = 1.7075$$

Which follow t-statistic with n-1=10 degrees of freedom

The table value of ‘t’ at 10 d.f for a one-tailed test at 5% level of significance is 1.812

Conclusion: since |t| =1.7075 < 1.812, the value of ‘t’ is not significant

We conclude that there is no significant difference in the marks of two tests before and after extra coaching, hence the students were not benefited by extra coaching.

Example2: A drug was administered to 10 patients and the increments in their blood pressure were recorded to be 6, 3, -2, 4, -3, 4, 6, 0, 3, and 2. Is it reasonable to believe that the drug has no effect on change of blood pressure'? Use 5% significance level.

Solution: It is given that n=10

Differences (increments) in blood pressure before and after administering the drug are

di= 6, 3, -2, 4, -3, 4, 6, 0, 3, 2.

we have to test

Null Hypothesis $H_0 : \mu_1 = \mu_2$ (*No significant difference in blood pressure*)

Alternate Hypothesis $H_1 : \mu_1 \neq \mu_2$ (*Two − tailed test*)(*There is a significant change in B.P*)

Level of significance $\alpha = 0.05$

$d_i=x_i-y_i$	d_i^2
6	36
3	9
-2	4
4	16
-3	9
4	16
6	16
0	0
3	9
2	4
23	**139**

$$\bar{d} = \frac{\sum d_i}{n} = \frac{23}{10} = 2.3$$

$$S = \sqrt{\frac{1}{n-1}\left(\sum_{i=1}^{n} d_i^2 - \frac{\left(\sum d_i\right)^2}{n}\right)}$$

$$= \sqrt{\frac{1}{9}\left[139 - \frac{(23)^2}{10}\right]} = \sqrt{9.5667} = 3.093$$

$$test\ statistic\ \ t = \left|\frac{\bar{d}}{S/\sqrt{n}}\right| = \left|\frac{2.3}{3.093/\sqrt{10}}\right| = 2.35$$

Which follow t-distribution with n-1=9 degrees of freedom

The table value of 't' at 9 d.f for a two-tailed test at 5% level of significance is ***2.262***

Conclusion: since |t| =2.35 > 2.262, the value of 't' is significant

We conclude that there is a significant difference in the Blood Pressure after administering the drug. i.e., there is some effect of drug on change of Blood Pressure.

4. F - test for EQUALITY OF POPULATION VARIANCES

TEST OF SIGNIFICANCE FOR RATIO OF VARIANCES

Suppose we want to test (i) whether two independent samples X_i, $(i = 1,2,.....n_1)$ and Y_j, $(j = 1,2,......n_2)$ have been drawn from the normal populations with the same variance σ^2 (say). **Or** (ii) whether the two independent estimates of the population variances are homogeneous or not.

Under the null hypothesis (Ho) that (i) $\sigma_1^2 = \sigma_2^2 = \sigma^2$ i.e., the population variances are equal or (ii) Two independent estimates of the population variances are homogeneous, the statistic F is given by

$$F = \frac{S_1^2}{S_2^2}$$

Where $$S_1^2 = \frac{1}{n_1 - 1}\sum_{i=1}^{n_1}(x_i - \bar{x})^2 \quad and \quad S_2^2 = \frac{1}{n_2 - 1}\sum_{j=1}^{n_2}(y_j - \bar{y})^2$$

are unbiased estimates of the common population variance σ^2 obtained from two independent samples and it follows Snedecor's F -distribution with (ν_1, ν_2) degrees of freedom. [Where $\nu_1 = n_1 - 1$ and $\nu_2 = n_2 - 1$].

Remarks: In F-statistic, greater of the two variances S_1^2 *and* S_2^2 is to be taken in the numerator and n_1 corresponds to the greater variance.

i.e., if $S_1^2 > S_2^2$ then $F = \frac{S_1^2}{S_2^2}$ follows $F(\nu_1, \nu_2)$ where $\nu_1 = n_1 - 1, \nu_2 = n_2 - 1$

if $S_2^2 > S_1^2$ then $F = \frac{S_2^2}{S_1^2}$ follows $F(\nu_1, \nu_2)$ where $\nu_1 = n_2 - 1, \nu_2 = n_1 - 1$

Conclusion: If the calculated value of 'F' is greater than the table value of $F(\alpha: \nu_1, \nu_2)$ then we reject the null hypothesis otherwise we accept it.

Note: The critical (rejection) values of 'F' for a two tailed test i.e.,

$$H_0 : \sigma_1^2 = \sigma_2^2 \ \ against \ \ H_1 : \sigma_1^2 \neq \sigma_2^2$$

are given by $F > F_{n_1-1, n_2-1}(\alpha/2)$ *and* $F < F_{n_1-1, n_2-1}(1-\alpha/2)$

Example1: Pumpkins were grown under two experimental conditions. Two random samples of 11 and 9 pumpkins show the sample standard deviations of their weights as 0.8 and 0.5 respectively. Assuming that the weight distributions are normal test the hypothesis that the true variances are equal at 5% level of significance.

Solution: Given that $n_1 = 11, n_2 = 9, \ S_1 = 0.8 \ \ and \ S_2 = 0.5$

We have to test $H_0 : \sigma_1^2 = \sigma_2^2$ against the alternative hypothesis

$H_1 : \sigma_1^2 \neq \sigma_2^2$

Level of significance α=0.05

The test statistic $F = \frac{S_1^2}{S_2^2} = \frac{(0.8)^2}{(0.5)^2} = \frac{0.64}{0.25} = 2.56$

The significant value of F at 5% level of significance F(10,8)= 3.35

Conclusion: The calculated value of F =2.56 < 3.35 , So there is no reason to reject the null hypothesis, Hence we accept H_0 and we conclude that there is no significant difference between the weights of the pumpkins which were grown under two experimental conditions with respect to their standard deviation of weights.

Example2: Two chemical companies can supply a raw material. The concentration of a particular element in this material is important. The mean concentration for both suppliers is the same, but we suspect that the variability in concentration may differ between the two companies. The standard deviation of concentration in a random sample of $n_1 = 10$ batches produced by company 1 is $S_1 = 4.7$ grams per liter, while for company 2, a random sample of $n_2 = 16$ batches yields $S_2 = 5.8$ grams per liter. Is there sufficient evidence to conclude that the two population variances differ? Use α=0.05.

Solution: Given that $n_1 = 10, n_2 = 16, \ S_1 = 4.7 \ \ and \ S_2 = 5.8$

We have to test $H_0 : \sigma_1^2 = \sigma_2^2$ against the alternative hypothesis

$H_1 : \sigma_1^2 \neq \sigma_2^2$

Level of significance α=0.05

$S_1^2 = (4.7)^2 = 22.09 \quad and \quad S_2^2 = (5.8)^2 = 33.64$

The test statistic $F = \frac{S_2^2}{S_1^2} = \frac{(5.8)^2}{(4.7)^2} = \frac{33.64}{22.09} = 1.5228$ $\left[\because S_2^2 > S_1^2\right]$

The significant value of F at 5% level of significance F(15,9)= 3.006

Conclusion: The calculated value of F =1.5228 < 3.006, so there is no reason to reject the null hypothesis, hence we accept H_0 and We conclude that the two variances are not differing.

Example3: The following table shows the yield of corn in bushels per plot in 20 plots, half of which are treated with phosphate as fertiliser.

Treated	5	0	8	3	6	1	0	3	3	1
Untreated	1	4	1	2	3	2	5	0	2	0

Test whether the treatment by phosphate has

(i) Reduced the variability of the plot yields, (ii) Improved the average yield of corn.

Solution: (i) Given that $n_1 = 10, n_2 = 10$

We have to test $H_0 : \sigma_1^2 = \sigma_2^2$ against the alternative hypothesis

$H_1 : \sigma_1^2 < \sigma_2^2 \quad or \quad \sigma_2^2 > \sigma_1^2$

Level of significance α=0.05

x_i	$\sum_{i=1}^{n_1}(x_i - \bar{x})^2$	y_j	$\sum_{j=1}^{n_2}(y_j - \bar{y})^2$
5	4	1	1
0	9	4	4
8	25	1	1
3	0	2	0
6	9	3	1
1	4	2	0
0	9	5	9
3	0	0	4
3	0	2	0
1	4	0	4
30	**64**	**20**	**24**

From the table we have

$\bar{x} = \frac{30}{10} = 3, \quad \sum_{i=1}^{n_1}(x_i - \bar{x})^2 = 64$

$S_1^2 = \frac{1}{n_1 - 1}\sum_{i=1}^{n_1}(x_i - \bar{x})^2 = \frac{1}{9}(64) = 7.11$

$\bar{y} = \frac{20}{10} = 2, \quad \sum_{j=1}^{n_2}(y_j - \bar{y})^2 = 24$

$S_2^2 = \frac{1}{n_2 - 1}\sum_{j=1}^{n_2}(y_j - \bar{y})^2 = \frac{1}{9}(24) = 2.67$

The test statistic $F = \frac{S_1^2}{S_2^2} = \frac{7.11}{2.67} = 2.66$ $\left[\because S_1^2 > S_2^2\right]$

The significant value of F at 5% level of significance F(9,9)= 3.18

Conclusion: The calculated value of F =2.66 < 3.18, so there is no reason to reject the null hypothesis, Hence, we accept H_0

and we conclude that there is no variability of plots before and after the treatment with phosphate as fertiliser.

** Solution for the question (ii) is left as assignment (Hint: we need to apply t-test for difference of means for independent samples)

5. χ^2- test for GOODNESS OF FIT

TEST OF SIGNIFICANCE BETWEEN OBSERVED AND THEORETICAL FREQUENCEIS

A very powerful test for testing the significance of the discrepancy between theory and experiment. The test is based on how good a fit we have between the frequency of occurrence of observations in an observed sample and the theoretical (expected) frequencies obtained from the hypothesized distribution.

If $O_i \ (i = 1,2,....n)$ is a set of observed (experimental) frequencies and $E_i \ (i = 1,2,....n)$ is the corresponding set of expected (theoretical or hypothetical) frequencies, then Karl Pearson's chi-square, given by

$$\chi^2 = \sum_{i=1}^{n} \left[\frac{(O_i - E_i)^2}{E_i} \right] \qquad provided \ \sum_{i=1}^{n} O_i = \sum_{i=1}^{n} E_i$$

Follows chi-square distribution with (n-1) degrees of freedom.

Conclusion: If the calculated value of χ^2 is greater than the table value of χ^2 at the level of significance α then we reject H_0 otherwise we accept H_0.

Example1: The following figures show the distribution of digits in numbers chosen at random from a telephone directory:

Digits	0	1	2	3	4	5	6	7	8	9	**Total**
Frequency	1026	1107	997	966	1075	933	1107	972	964	853	**10,000**

Test whether the digits may be taken to occur equally frequently in the directory.

Solution: Given that n=10

Null Hypothesis H_0: All the digits occur equally frequently in the directory (i.e., each digit is repeated equal number of times)

$$\left[the\ Expected\ Frequecny\ for\ each\ digit\ of\ 0,1,2,.....9\ is\ E_i = \frac{10{,}000}{10} = 1000 \right]$$

Alternate Hypothesis H_1: All the digits do not occur equally frequently.

Level of significance : 0.05

CALCULATIONS FOR χ^2

Digit	O_i	E_i	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
0	1026	1000	676	0.676
1	1107	1000	11449	11.449
2	997	1000	9	0.009
3	966	1000	1156	1.156
4	1075	1000	5625	5.625
5	933	1000	4489	4.489
6	1107	1000	11449	11.449
7	972	1000	784	0.784
8	964	1000	1296	1.296
9	853	1000	21609	21.609
	10000	**10000**		**58.542**

Test statistic : $\chi^2 = \sum_{i=1}^{n}\left[\frac{(O_i - E_i)^2}{E_i}\right]$ provided $\sum_{i=1}^{n} O_i = \sum_{i=1}^{n} E_i$

$$\chi^2 = \sum_{i=1}^{n}\left[\frac{(O_i - E_i)^2}{E_i}\right] = 58.542$$

The number of degrees of freedom (n-1) = 10 - 1 = 9
The tabulated value of $\chi^2_{0.05} = 16.919$

Conclusion: Since the calculated value of χ^2 is much greater than the tabulated value, it is highly significant, and we reject the null hypothesis.
Thus, we conclude that the digits are not uniformly distributed in the directory.

Example2: The theory predicts the proportion of beans in the four groups A, B, C and D should be 9 : 3 : 3 : 1. In an experiment among 1600 beans the numbers in the four groups were 882, 313, 287 and 118. Does the experimental result support the theory? (At α=0.05)

Solution: Given that n=10
Null Hypothesis H_0 : The theory fits well into the experiment. i.e., the experimental results support the theory that the beans are in the proportion of 9:3:3:1.
Alternate Hypothesis H_1 : The proportion of beans are not in the given proportion.
Level of significance : 0.05

Test statistic : $\chi^2 = \sum_{i=1}^{n}\left[\frac{(O_i - E_i)^2}{E_i}\right]$ provided $\sum_{i=1}^{n} O_i = \sum_{i=1}^{n} E_i$

Under the null hypothesis, the expected (theoretical) frequencies are computed as follows:
Total number of beans = 882 + 313 + 287 + 118 = 1600
These are to be divided in the ratio 9 : 3 : 3 : 1
E(882) = (9/16) X 1600 = 900, E(313) = (3/16) X 1600 = 300 ,
E(287) = (3/16) X 1600 = 300, E(118) = (1/16) X 1600 = 100

CALCULATIONS FOR χ^2

O_i	E_i	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
882	900	324	0.36
313	300	169	0.5633
287	300	169	0.5633
118	100	324	3.24
1600	**1600**		**4.7267**

$$\chi^2 = \sum_{i=1}^{n}\left[\frac{(O_i - E_i)^2}{E_i}\right] = 4.7267$$

The number of degrees of freedom (n-1) = 4 - 1 =3
The tabulated value of $\chi^2_{0.05} = 7.815$

Conclusion: Since the calculated value of $\chi^2 = 4.7267 < 7.815$, it is not significant, and we cannot reject the null hypothesis. Hence, we may accept H_0 at 5% level of significance.
Thus, we conclude that there is good correspondence between theory and experiment.

6. χ^2- test for INDEPENDENCE OF ATTRIBUTES

TEST OF SIGNIFICANCE BETWEEN OBSERVED AND THEORETICAL FREQUENCEIS

The chi-squared test procedure can also be used to test the hypothesis of independence of two attributes of classification.

Let us consider two attributes A and B. A divided into 'r' classes and B divided into 'c' classes, such a classification in which attributes are divided into more than two classes is known as manifold classification. The various cell frequencies can be expressed in the table known as 'r x c' manifold contingency table. The row and column totals in the Table are called marginal frequencies.

Our decision to accept or reject the null hypothesis, 'Ho' of independence between the two attributes is based upon how good a fit we have between the observed frequencies and its Expected frequencies.

Under **Ho** the Expected frequencies can be calculated for each cell value of the observed frequencies (O_{ij}) by the relation

$$E(O_{ij}) = E_{ij} = \frac{(A_i)(B_j)}{N} = \frac{row\ total \times Column\ total}{Grand\ Total}$$

Then the chi-square statistic is given by

$$\chi^2 = \sum_{i=1}^{r}\sum_{j=1}^{c}\left[\frac{(O_{ij}-E_{ij})^2}{E_{ij}}\right] = \sum_{i=1}^{n}\left[\frac{(O_i-E_i)^2}{E_i}\right] \qquad provided\ \sum O_{ij} = \sum E_{ij}$$

Which follows χ^2 distribution with $[(r-1)\times(c-1)]$ degrees of freedom.

Where r=number of rows, c=number of columns.

Conclusion: If the calculated value of χ^2 is greater than the table value of χ^2 at the level of significance α then we reject H_0 otherwise we accept H_0.

Example1: A company has to choose among three pension plans. Management wishes to know whether the preference for plans is independent of job classification and wants to use α= 0.05. The opinions of a random sample of 500 employees are given in the following table

		Pension plans			
		1	2	3	Total
Job classification	Salaried workers	160	140	40	**340**
	Hourly workers	40	60	60	**160**
	Total	**200**	**200**	**100**	**500**

Test whether the selection of Pension plans independent of Job classification.

Solution: Given that r=2 and c=3 and N=500

Null Hypothesis Ho: Selection of Pension plans independent of Job classification.

Alternate Hypothesis H_1 : Both are not independent

Level of Significance **α** : 0.05

CALCULATION OF CHI-SQUARE VALUE

O_i	E_i	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
160	$\frac{340\times200}{500}=136$	576	4.2353
140	$\frac{340\times200}{500}=136$	16	0.1176
40	$\frac{340\times100}{500}=68$	784	11.5294
40	$\frac{160\times200}{500}=64$	576	9
60	$\frac{160\times200}{500}=64$	16	0.25
60	$\frac{160\times100}{500}=32$	784	24.5
500	**500**		**49.6323**

$$\chi^2 = \sum_{i=1}^{n}\left[\frac{(O_i - E_i)^2}{E_i}\right] = 49.6323$$

Degrees of freedom =[(r-1)X(c-1)]
=[(2-1)X(3-1)]=2

The tabulated value of $\chi^2_{0.05} = 5.99$

Conclusion: Since the calculated value of $\chi^2 = 49.6323 > 5.99$, it is highly significant and we reject the null hypothesis.

Thus we conclude that the preference for pension plans is not independent of job classification.

Example2: The following data is for a sample of 300 car drivers who were classified with respect to age and the number of accidents they had during past two years. Test whether there is any relationship between the age of drivers and the no. of accidents they had.

		No. of accidents			
		0	***1 or 2***	***3 or more***	
Age of drivers	***≤ 21***	8	23	14	45
	22-26	21	42	12	75
	≥ 27	71	90	19	180
		100	155	45	300

Solution: From the given data we observe that
Number of rows r=3, Number of columns c=3
Null Hypothesis H_0 : Age of drivers and number of accidents are independent.
Alternate Hypothesis H_1: Number of accidents depends on age of drivers.
Level of significance α=0.05

$$E(8) = \frac{45\times100}{300} = 15; \quad E(23) = \frac{45\times155}{300} = 23.25;$$

$$E(21) = \frac{75\times100}{300} = 25; \quad E(42) = \frac{75\times155}{300} = 38.75$$

CALCULATION OF χ^2 value

O_i	E_i	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
8	15	49	3.266667
21	25	16	0.64
71	60	121	2.016667
23	23.25	0.063	0.002688
42	38.75	10.56	0.272581
90	93	9	0.096774
14	6.75	52.56	7.787037
12	11.25	0.563	0.05
19	27	64	2.37037
		TOTAL	**16.50278**

$$\chi^2 = \sum_{i=1}^{n}\left[\frac{(O_i - E_i)^2}{E_i}\right] = 16.50278$$

The calculated values of | χ^2 |=16.50278
Table value (significant value) of Chi-square at [(r-1)(c-1)]= (2X 2)=4 degrees freedom
With 5% level of significance is

$$\chi^2_{0.05} = 9.488$$

Conclusion: the calculated value of | χ^2 | = 16.50278 > 9.488 (Z table value), it is highly significant. Hence we conclude that the number of accidents and age of drivers are not independent. i.e., the number of accidents depends on the age of drivers.

PRACTICE QUESTIONS

1. The average length of time for students to register for summer classes at a certain college have been 50 minutes. A new registration procedure using modern computing machines is being tried. If a random sample of 12 students had an average of registration time 42 minutes with standard deviation of 11.9 minutes under new system, Test the hypothesis that the new system significantly reduces the registration time at 0.05 level of significance.

2. A manufacturer of gunpowder has developed a new powder which is designed to produce a muzzle velocity equal to 3000 ft/sec. A sample of 7 shells is loaded with the charge and the muzzle velocities measured. The resulting velocities are as follows: 3005, 2935, 2965, 2995, 3105, 2935 and 2905. Do these data present sufficient evidence to indicate that the average velocity differs from 3000 ft/sec?

3. Two suppliers manufacture a plastic gear used in a laser printer. The impact strength of these gears measured in foot-pounds is an important characteristic. A random sample of 10 gears from supplier 1 results in $\overline{x_1} = 290$ *and* $s_1 = 12$ while another random sample of 16 gears from the second supplier results in $\overline{x_2} = 321$ *and* $s_2 = 22$. Is there evidence to support the claim that supplier 2 provides gears with higher mean impact strength? Use $\alpha = 0.05$

4. It is believed that glucose treatment will extend the sleep time of mice. In an experiment to test this hypothesis 10 mice selected at random are given glucose treatment and are found to have a mean hexobarbital sleep time of 47.2 min with a standard deviation of 9·3 min. A further sample of 12 untreated mice are found to have a mean hexobarbital sleep time of 28·5 min. with a standard deviation of 7·2 min. Are these results significant evidence in favour of the hypothesis?

5. The scores of 10 candidates prior and after training are given below:

Prior	84	48	36	37	54	69	83	96	90	65
After	90	58	56	49	62	81	84	86	84	75

Test whether the training effective at 0.01 level of significance.

6. The following table gives the additional hours of sleep gained by 10 patients in an experiment to test the effect of a drug. Do these data give evidence that the drug produces additional hours of sleep? Use 0.01 level of significance.

Patients	1	2	3	4	5	6	7	8	9	10
Hours Gained	0.7	0.1	0.2	1.2	0.31	0.4	3.7	0.8	3.8	2

7. A research was conducted to understand whether women have a greater variation in attitude on political issues than men. Two independent samples of 31 men and 41 women were selected for the study. The sample variances so calculated were 120 for women and 80 for men. Test whether the difference in attitude toward political issues is significant at 5 percent level of significance. ($H_1 : \sigma_w^2 > \sigma_m^2$)

8. The following data relate to the number of units of an item produced per shift by two workers A and B for a number of days

A	19	22	24	27	24	18	20	19	25		
B	26	37	40	35	30	30	40	26	30	35	45

(i) Is there any significant difference between the average number of units produced by the two workers (use α=0.05)
(ii) Can it be inferred that worker A is more stable compared to worker B? (Use α=0.05)

9. The grades in an Engineering course for a particular semester were as follows

Grade	A	B	C	D	E
Frequency	14	18	32	20	16

Test the hypothesis, at 0.05 level of significance, that the distribution of grades is uniform.

10. A bird watcher sitting in a park has spotted a number of birds belonging to 6 categories. The exact classification is given below.

Category	1	2	3	4	5	6
Frequency	6	7	13	17	6	5

Test at 5% level of significance whether or not the data is compatible with the assumption that this particular park is visited by birds belonging to these six categories in the proportion 1 : 1 : 2 : 3 : 1 : 1.

11. A random sample of students is asked their opinions on a proposed core curriculum change. The results are as follows.

		OPINION	
		Favouring	Opposing
CLASS	Freshman	120	80
	Sophomore	70	130
	Junior	60	70
	Senior	40	60

Test at α=0.05, the hypothesis that opinion on the change is independent of class standing.

12. In an experiment to study the dependence of hypertension on smoking habits, the following data were taken on 180 individuals:

	Non-Smokers	Moderate Smokers	Heavy Smokers
Hypertension	21	36	30
No Hypertension	48	26	19

Test the hypothesis that the presence or absence of hypertension is independent of smoking habits. Use 0.05 level of significance.

UNIT-4

SUMMARY

1. t-test for Single Mean: To test the significance difference between the sample mean $\bar{x}$ and the specified population mean μ the test statistic is given by $t = \dfrac{\bar{x} - \mu}{S/\sqrt{n}}$

2. t-test for difference of Means: To test the significant difference between two means, the test statistic is given by

$$t = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{S^2\left[\frac{1}{n_1} + \frac{1}{n_2}\right]}}$$

3. Paired t-test (or) t-test for difference of Means for dependent samples: To test the significant difference between the means of dependent sample

The test statistic is $$t = \frac{\bar{d}}{S/\sqrt{n}}$$

4. F-test for equality of variances: To test whether two independent samples $X_i, (i = 1,2,......n_1)$ and $Y_j, (j = 1,2,......n_2)$ have been drawn from the normal populations with the same variance σ^2

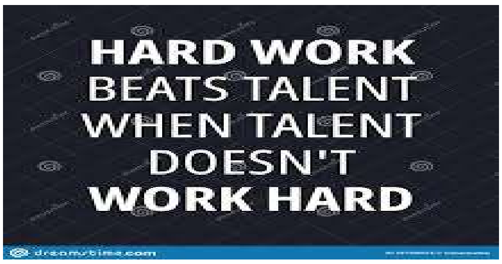
the test statistic is $$F = \frac{S_1^2}{S_2^2}$$

5. χ^2- test for GOODNESS OF FIT: To test the significant difference between the observed frequencies and the expected (hypothetical) frequencies the chi-square test statistic is defined as

$$\chi^2 = \sum_{i=1}^{n}\left[\frac{(O_i - E_i)^2}{E_i}\right]$$

6. χ^2- test for INDEPENDENCE OF ATTRIBUTES: To test the association (or) independence between two attributes, the test statistic is

$$\chi^2 = \sum_{i=1}^{r}\sum_{j=1}^{c}\left[\frac{(O_{ij} - E_{ij})^2}{E_{ij}}\right] = \sum_{i=1}^{n}\left[\frac{(O_i - E_i)^2}{E_i}\right]$$



"Success is no accident. It is hard work, perseverance, learning, studying, sacrifice and most of all, love of what you are doing or learning to do."